Comparison (Vorpal vs. formulas) of transverse and longitudinal friction force coefficients

(December 15, 2003)

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First step in cooling dynamics studies is to choose reliable formula for the friction force.

In principle, all available formulas agree within a factor of 3 for typical parameters.

However, for some parameters, on get significant differences between longitudinal and transverse cooling, depending on which formulas are used. Also, for some parameters, different formulas can give completely different behavior.

The Vorpal codes is used to benchmark various formulas in different regimes of parameters.

Basic sets of formulas

Derbenev-Skrinsky (D-S) - analytic

$$F_\parallel^A = -\frac{3}{2}\omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0}\ln\left(\frac{\rho_{max}^A}{\rho_{min}^A}\right)\left(\frac{V_\perp}{V_{ion}}\right)^2 \frac{V_\parallel}{V_{ion}^3}$$

$$F_\perp^A = -\frac{1}{2}\omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0}\ln\left(\frac{\rho_{max}^A}{\rho_{min}^A}\right)\frac{(V_\perp^2 - 2V_\parallel^2)}{V_{ion}^2}\frac{V_\perp}{V_{ion}^3}$$

$$\rho_{max}^A = \min(r_{beam}, \rho_{max})$$

$$\rho_{min}^A = \max(r_L, \rho_{min})$$

$$r_L = V_{\perp,RMS,e}/\Omega_L(B_\parallel)$$

Derbenev-Skrinsky-Meshkov (D-S-M) - analytic

$$F_\parallel^A = -\frac{3}{2}\omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0}\left(\ln\left(\frac{\rho_{max}^A}{\rho_{min}^A}\right)\left(\frac{V_\perp}{V_{ion}}\right)^2 + 2/3\right)\frac{V_\parallel}{V_{ion}^3}$$

Factor 2/3 without Ln offsets "defect" of adiabatic collisions by contributions with large impact parameters so that integral momentum transfer is no longer zero in long. direction when V_tr=0

V. Parkhomchuck (VP) - empiric

$$\mathbf{F} = -\frac{1}{\pi}\omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0}\ln\left(\frac{\rho_{max} + \rho_{min} + r_L}{\rho_{min} + r_L}\right)\frac{\mathbf{V}_{ion}}{(V_{ion}^2 + V_{eff}^2)^{3/2}}$$

First scaled regime

"Scaled-1" regime:

- **First studies with "scaled-1" RHIC parameters (reported by D. Bruhwiler, August 2003).**

- **First results showed that D-S formulas overestimate cooling force, the longitudinal friction coefficient was found in good agreement with VP formula**

- **Disagreement with VP formula for transverse cooling force was attributed to bulk space charge force**

   At BNL, we attempted to study dependence of friction force on ion velocity:

- **Disagreement with VP formula for longitudinal cooling as a function of ion velocity was unclear.**

- **Subsequent study on velocity dependence using analytic formulas showed that we are actually in a different region on the Force vs Velocity diagram than expected.**

- **As a result, classical statement of where one should expect the max of cooling force were revisited – since maximum was observed at very different place! – will be discussed tomorrow**

"scaled-1" regime

Electron bunch

$$V_{\perp,RMS,e} = 5x10^5 \, m/s$$

$$V_{\parallel,RMS,e} = 1x10^3 \, m/s$$

$$r_{\perp,RMS,e} = 1x10^{-4} \, m$$

$$r_{\parallel,RMS,e} = 1x10^{-3} \, m$$

$$n_e = 6.35x10^{14} \, m^{-3}$$

$$\omega_{pe} = 1.4x10^9 \, rad/s$$

Single Au+79 ion

$$V_{\parallel} = 5x10^4 \, m/s$$

$$V_{\perp} = variable$$

$$Z = 5*79$$

$$\rho_{max}^A = \rho_{max} = 3.5x10^{-5} \, m$$

$$\rho_{min} = 2.7x10^{-6}$$

$$\rho_{min}^A = r_L = 2.8x10^{-6} \, m$$

System parameters

$$B_{\parallel} = 1 \, Tesla$$

$$L = 30 \, m$$

$$\tau = (L/\gamma\beta c) = 9.35x10^{-10} \, s$$

Coulomb logarithms

$$\ln\left(\frac{\rho_{max} + \rho_{min} + r_L}{\rho_{min} + r_L}\right) \approx 2$$

$$\ln\left(\frac{\rho_{max}^A}{\rho_{min}^A}\right) \approx 2.8$$

ρ_min on previous slide was calculated using Z=79 while scaled parameter used in simulation was Z=5*79 so that ρ_min was actually $13.5*10^{-6}$ m.

As a result, Cooling Log was much smaller and we happened to be seating at the region were behavior of cooling force is changing dramatically.

This is not "RHIC-regime" but, in fact, it is a very interesting regime to benchmark formulas. It also led to some interesting findings.

Calculated F_{cool} based on VP formula for "scaled-1" parameters used in Vorpal simulations

F_{cool}
[in normalized units]

we are here

V_{ion} [m/sec]
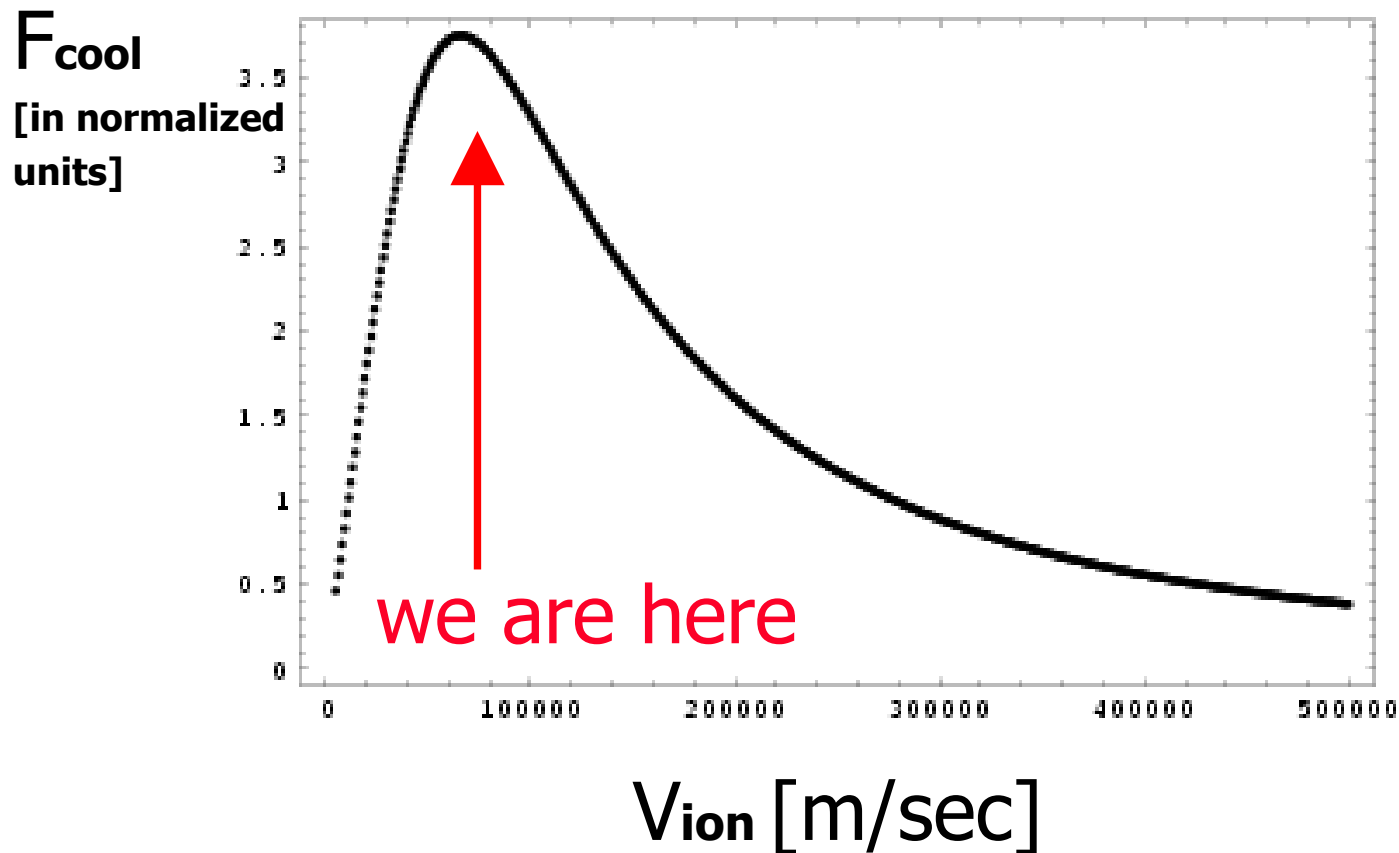
Comparison of $\delta v\_parallel$ between VP formula and numerical calculations using Vorpal code.
Table 1

	δv_parallel using VP formula with correct impact parameters	Vorpal calculation for a single random seed numbers
V_tr= V_parallel=12000 m/s	-3.1	-3.5
V_tr=V_parallel=25000m/s	-6.0	-6.5
V_tr=V_parallel=50000 m/s	-8	-10
V_tr=V_parallel=Sqrt[8]*50000 m/s	-6.1	-5

Table 1 - comments

The goal was to check dependence on V_ion:

We can see a very good agreement between VP and Vorpal even for the most difficult region near the maximum of cooling Force.

Note that maximum of cooling force does not happen in this case at $\Delta e\_parallel$ as one can find in the literature on electron cooling theory – will be discussed later

Removing bulk space-charge force

- **Previous studies of transverse friction force in direct numerical simulations using Vorpal code showed strong dependence on bulk space-charge force.**

This contribution of space-charge force was removed by doing additional runs with negative ions and then finding the average.

As a result, one also gets agreement between transverse friction force in VP formula and direct numerical calculation using Vorpal.

Comparison of δv_transverse between VP formula and numerical calculations using Vorpal code.
Table 2

	δv_transverse using VP formula with correct impact parameters	Vorpal calculation for a single random seed numbers
V_tr= V_parallel=12000 m/s	-3.1	-3.8
V_tr=V_parallel=25000m/s	-6.0	-5.1
V_tr=V_parallel=50000 m/s	-8	-6
V_tr=V_parallel=Sqrt[8]*50000 m/s	-6.1 -5.5(old)	-3.6 -57(old)

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Friction force for RHIC parameters

We are here
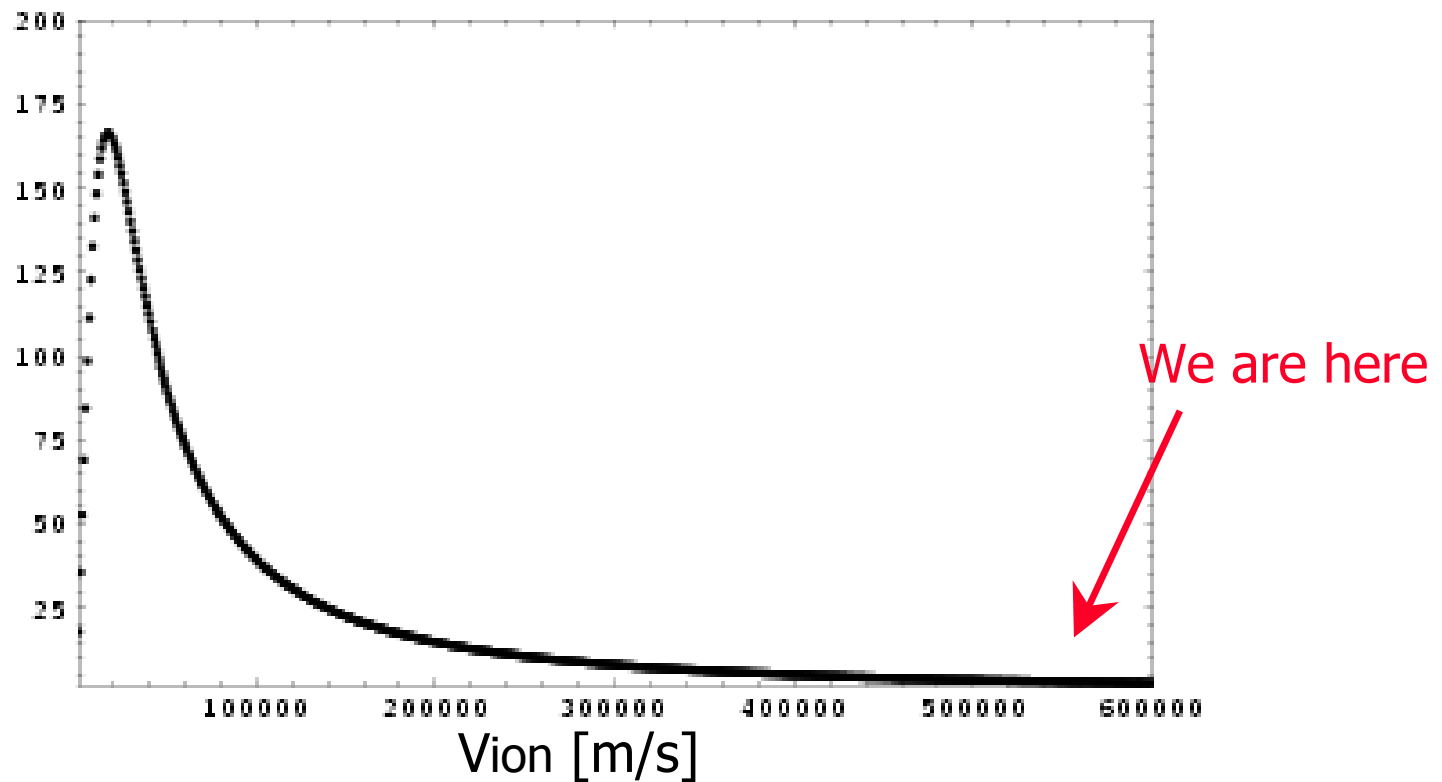
RHIC parameters vs "scaled-1" Vorpal runs

	Vorpal	RHIC
Vion_parallel [m/s]	$5*10^4$	$3*10^5$
Vion_transverse [m/s]	$7*10^4$	$6*10^5$
Zion	$5*79$	79
Ve_parallel [m/s]	$1*10^3$	$9*10^4$
Ve_transverse [m/s]	$5*10^5$	$9*10^6$
σ_x [m]	0.0001	0.0015
σ_z [m]	0.001	0.05
n_e [m-3]	$6.35*10^{14}$	$2.7*10^{15}$
ω_{pe} [rad/s]	$1.4*10^9$	$2.9*10^9$

Scaling to RHIC parameters

Several approaches:

1. **Requires large number of particles:**

   Scale everything back to RHIC parameters also increasing transverse rms beam sizes by a factor of 10 each. To keep the same n_e requires going to 400*100K -> 40M electrons in simulation – possible on parallel computer at NERSC.

2. **Small number of particles:**

   2.1 Can scale various impact parameters accordingly to reproduce situation similar to RHIC – should be similar physics – was used to study RHIC-regime here at BNL.

   2.2 Simple scaling factor in front of the force expression. With this scaling factor in mind, we can do simulation for realistic RHIC regime on a single CPU –dependence of "electron charge" scaling was studied.

   2.3 Can reduce the problem assuming 1-D trasverse velocities, uniform density, etc. – was used at Tech-X (will be reported by D. Bruhwiler)
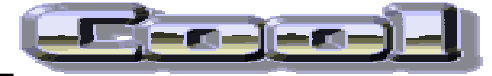
- **New scaling was used to study/benchmark cooling force for non-magnetized cooling – will be reported by D. Bruhwiler et al.**

- **Scaling based on impact parameters was used to study**

1. **"RHIC regime" - tomorrow.**

2. **As well, as to study condition of "bad magnetization" -tomorrow including different dependence of friction force on V_ion and electron velocities.**

3. **Maximum of the force – tomorrow.**

- **Some numerical tests and studies – like dependence on number of particles, "electron charge" scaling, etc. were also done and can be discussed later.**